

26. MATHEMATICS

Preface

The programme aims to lay a strong basic foundation for higher mathematics both in pure and applied branches of Mathematics. It is meant for students who wish to pursue their careers involving mathematical research and skills. The programme is intended to teach the students the art of problem solving activities in both branches of Mathematics.

Programme Outcomes:

At the end of the course, students:

1. Will have a strong foundation in both the pure and applied Mathematics.
2. Will be able to ask logical questions and also be able to solve them.
3. Will be able to interact with people from outside the state and communicate their ideas effectively.
4. Will have a sound knowledge in programming and computation.

MTH-100: FUNDAMENTAL MATHEMATICS-I

(Contact Hours: 60, Credits: 4)

Objectives: The primary objective of this course is to introduce the foundational concepts of calculus and techniques of problem. The students will also learn the methods of classical algebra and the art of solving a cubic equation.

Course Outcomes : After this course students will be able to learn the rigorous idea of limit of a function which is foundational to grasp the concepts of continuity, differentiation. In addition to this the students will be able to calculate the volume and surface area of solids of revolution and learn the applications of continuity and derivative in Higher Algebra.

Unit I : Limit and Continuity
hours)

(15

ϵ - δ definition of limit of a real valued function; standard theorems; limit at infinity and infinite limits; ϵ - δ definition of continuity of a real valued function; standard theorems; geometrical interpretation of continuity; discontinuity - types of discontinuity; properties of continuous functions; Intermediate Value Theorem and its applications; fixed point theorem; location of roots - theorem and its application.

Unit II : Differentiability

(15 hours)

Differentiability of a real-valued function of a real variable; geometrical significance; standard theorems; stationary point; local extrema; Rolle's Theorem, Lagrange's Mean Value Theorem, Cauchy's Mean Value Theorem and their applications; differentiability and monotonicity; concavity; inflection point; differential; successive differentiation; Leibnitz's Theorem.

Unit III : Integral Calculus

(15 hours)

Definite Integral as a limit of a sum; fundamental theorem of integral calculus; properties of definite integral; applications of definite integral - area under a curve, length of simple plane curves, volume and surface areas of solids of revolution in standard cases; reduction formulas for $\int \sin^n x dx$, $\int \cos^n x dx$, $\int \tan^n x dx$, $\int e^{ax} x^n dx$, $\int x^n \log x^n dx$, $\int \sin^n x \cos^m x dx$.

Unit IV : Complex Numbers & Theory of Equations

(15

hours)

Complex Numbers - properties; polar representation; Polynomials over $\mathbb{Z}, \mathbb{Q}, \mathbb{R}, \mathbb{C}$ - definition and standard properties; Division Algorithm; gcd, Euclidean Algorithm, Unique Factorisation Theorem over $\mathbb{Q}, \mathbb{R}, \mathbb{C}$ (statement and application); root of a polynomial; detailed study of the roots of a polynomial; Fundamental Theorem of Algebra (statement and corollary) and its failure over $\mathbb{Z}, \mathbb{Q}, \mathbb{R}$; Remainder Theorem and Factor Theorem; Synthetic division; multiple roots; complex roots and surd roots; Descartes' rule of signs; Relation between roots and coefficients of a polynomial; symmetric functions of roots with special reference to cubic equations; n^{th} roots of unity; De Moivre's Theorem and its applications; Euler's Theorem (statement only); solution of a cubic equation by Cardan's Method.

Notes: A candidate must obtain the minimum pass marks (as per NEHU Rule) to clear the course.

Suggested Readings:

1. Calculus, H. Anton, I. Bivens, S. Davis, Wiley India Pvt. Ltd. (2015).
2. Differential Calculus, R.K. Ghosh, K.C. Maity, New Central Book Agency Ltd. (2011).
3. Integral Calculus, R.K. Ghosh, K.C. Maity, New Central Book Agency Ltd. (2013).
4. Higher Algebra Classical, S.K. Mapa, Levant Books India (2021).
5. Mathematical Analysis, S.C.Malik, S.C.Arora, New Age International Publication (2021).
6. Thomas Calculus, G.B.Thomas, J. Hass, C. Heil, Pearson Education (2018).
7. Calculus: Early Transcendentals, J. Stewart, Cengage India Pvt Ltd. (2017).
8. Introduction to Real Analysis, R.G. Bartle, D. R. Sherbert, Wiley India Edition (2021).
9. Higher Algebra, B.Das, S.R.Maity, AsokePrakasan (2010).

MTH-150: FUNDAMENTAL MATHEMATICS-II

(Contact Hours: 60, Credits: 4)

Learning Objectives: The primary objective of this course is to study the properties of standard geometrical objects in two and three dimensional spaces. The course will also introduce the basic concepts of multivariable calculus and vector calculus with applications in Physics.

Unit I : Two Dimensional Geometry (15 hours)

Transformation of coordinates - Change of axes, invariants, removal of xy term.

Pair of straight lines - General and homogeneous equations of second degree, angles between pair of straight lines represented by a second degree equation, bisectors of the angles between a pair of straight lines through the origin.

Conics - General equation of second degree, reduction to standard form, equation of tangents, conditions of tangency, equation of normal, parametric form of conics, conjugate diameters of ellipse and hyperbola.

Unit II : Three Dimensional Geometry (15 hours)

Planes - General equation of a plane, normal form of a plane, angle between two planes, perpendicular distance of a point from a plane, planes through intersection of two planes.

Spheres - General equation of a sphere, plane section of a sphere, sphere through a given circle, tangent plane, intersection of two spheres.

Cones - Equation of a cone with a conic as a guiding curve, enveloping cone, mutually perpendicular generators, tangent planes, reciprocal cone, right circular cone.

Unit III : Multivariable Differential Calculus (15 hours)

Real-valued functions of two and three real variables ($f: \mathbb{R}^2 \rightarrow \mathbb{R}, f: \mathbb{R}^3 \rightarrow \mathbb{R}$); Limits and continuity of real-valued functions of two and three real variables (basic concepts and simple problems); Partial Derivatives of first order and its geometrical significance.

Second order partial derivatives - basic concepts and examples; Schwarz's theorem (statement and examples only); Laplacian; Chain rule; Euler's theorem on homogeneous functions upto three variables.

Unit IV : Vector Calculus

(15 hours)

Scalar and vector products of three and four vectors - properties, geometrical significance, and applications.

Vector-valued functions of real variables ($f: \mathbb{R} \rightarrow \mathbb{R}^2, f: \mathbb{R} \rightarrow \mathbb{R}^3$); Derivative of a vector-valued function of a real variable; Properties and geometrical applications - arc length, unit tangent vector, normal vector, curvature.

Gradients of real-valued functions of two or three variables - physical and geometrical significance, and elementary properties; Directional derivatives of real-valued functions of two or three variables and its geometrical significance, maximum directional derivative; Tangent planes and normal lines.

Divergence & Curl - physical and geometrical significance, and elementary properties; Solenoidal and irrotational vector fields.

Course Outcomes :After this course students will be able to understand the properties of geometrical objects in two and three dimensions. They will learn conceptual variations while advancing from one variable to several variable in calculus. In addition to this they will intuitively understand how the language of vectors is used in other fields of science like Physics.

Notes: A candidate must obtain the minimum pass marks (as per NEHU Rule) to clear the course.

Suggested Readings:

1. Calculus, H. Anton, I. Bivens, S. Davis, Wiley India Pvt. Ltd. (2015).
2. Analytical Geometry and Vector Analysis, B. Das, Orient Book Company (2008).
3. Analytical Solid Geometry, S. Narayan, P.K. Mittal, S. Chand & Company (2007).
4. Differential Calculus, R.K. Ghosh, K.C. Maity, New Central Book Agency Ltd. (2011).
5. Vector Analysis, R.K. Ghosh, K.C. Maity, New Central Book Agency Ltd. (2011).
6. Analytical Geometry and Vector Analysis, J.G. Chakravorty, P.R. Ghosh, U.N. Dhur& Sons Pvt Ltd. (2012).
7. Calculus: Early Transcendentals, J. Stewart, Cengage India Private Limited. (2017).
8. Calculus and Analytic Geometry, G.B. Thomas Jr., R.L. Finney, Pearson Education India (2010).
9. Vector Calculus, S. J. Colley, Pearson (2012).
